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groups which play such an important rôle in the science of our epoch."* "The group concept was employed in the preceding century (about 1770) simultaneously by Lagrange and Vandermonde, and since this time it occupies a prominent place in the theory of algebraic equations. In regard to this it is only necessary to refer to the name of Galois. Hence group theory has been regarded as a supplement of algebra. This however is incorrect. For the group concept extends far beyond this into almost all other parts of mathematics."†

"It was reserved for Galois to place the theory of equations on a definite foundation by showing that to each equation there corresponds a substitution group in which are exhibited its essential characteristics, and especially those which relate to its solution by other auxiliary equations."‡ "I should reproach myself for forgetting, even in so rapid a resumé, the applications which Lie has made of his theory of groups to the non-Euclidean geometry and to the profound study of the axioms which lie at the basis of our geometric knowledge."§

"The theory of groups, which is making itself felt in nearly every part of higher mathematics, occupies the foremost place among the auxiliary theories which are employed in the most recent function theory."‖ "It need scarcely be added that some modern mathematicians seem to avoid group theory even where it would simplify the treatment of the subject in hand. This seems to be true, for instance, of Hilbert's Grundlagen der Geometrie."¶

Leland Stanford University.

OUR SYMBOL FOR ZERO.

By DR. GEORGE BRUCE HALSTED.

At the Paris International Congress, the paper of my erudite friend of Japanese days, Professor Fiyisawa, attracted, I believe, apart from the great address of Hilbert, more favorable attention than any other.

In praising this paper, I ventured there to emphasize, that the appearance in Japan, before any communication with Europe, of a positional notation for number with precisely the symbol for zero which we now use, and which, as Professor Cajori says in the February number of the American Mathematical Monthly, has been supposed of Hindu origin, raised the question for the future historian of mathematics of the relation or connection between these two indubitable, however widely separated, appearances of the same peculiar symbol.

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* Picard, *Traite d' analyse*, Vol. 3, 1896, page 492.

† Klein, *Einleitung in die hoehere Geometrie* II, 1893, page 3.

‡ Jordan, *Traite des substitutions*, 1870, Preface.

§ Darboux, *Comptes Rendus*, Vol. 128, 1899, page 528.

‖ Fricke und Klein, *Automorphe Functionen*, Vol. 1, 1897, page 1.

¶ Poincare, *Bulletin des Sciences Mathematiques*, Vol. 26, 1902, page 272.

Professor M. Cantor, who was presiding over the section, seemed perturbed, perhaps at the very idea of future historians of mathematics, and strenuously insisted that the idea of positional notation for number and the symbolization of zero should be accredited to the Babylonians. Of course we know that there are some fragmentary matters which point to the Babylonians using sixty as a number radix, but I have never learned that any Babylonian symbol for zero has been discovered, and the question to which I called attention remained untouched. But now, if we may accept the testimony of Y. Mikami, of Tokio, one of those questions has been answered. He says, "I have found very important relations between the mathematics of India and of China." But we know that Japanese hieroglyphic writing, like Japanese Buddhism, came through China.

Therefore the appearance of our symbol for zero in Hindustan and Japan is not one of the mysterious and seemingly inexplicable coincidences, such as the prehistoric appearance and importance of the cross in the pre-Columbian religions of Yucatan and Mexico. For here we find a water-way connecting Hindustan and Japan through China. But the source of the flow still remains undetermined.

Use of the abacus might have been what suggested the symbolization of zero. But Hindustan from time immemorial used systems of abacal calculation. Greece, and even mathematically stupid Rome, had the abacus.

Every claim for importance in the world of ideas hitherto made for China has evaporated into nothingness. They did not even invent gunpowder or the mariner's compass. Remember their exaltation and subsequent,downfall in the history of astronomy.

So it will be as to their claim to our zero symbol. They got it, as they did their Buddhism, from India, and passed on both to Japan.

ANNAPOLIS, MD., *March 7, 1903.*

HARMONIC PAIRS IN THE COMPLEX PLANE.

A PURELY GEOMETRICAL TREATMENT FOR CERTAIN MAPS DEFINED BY THE SUBSTITUTION $w=\frac{1}{2}(z+\frac{1}{z})$.

By ARCHIBALD HENDERSON. Ph. D.. Associate Professor of Mathematics,
University of North Carolina. Chapel Hill.

§1. A subject of especial interest in the Theory of Functions is the conception of harmonic pairs in the complex plane. The proof given below offers initially a direct interpretation of the equation

$$\left|\frac{z-z_1}{z-z_2}\right|=\rho\ldots.(1),$$